BT-MRE-I&II-05.18-1018 Reg.No. [ ] **A**

B.Tech. Degree I&II Semester Examination in Marine Engineering May 2018

MRE 1101 ENGINEERING MATHEMATICS I

(2013 Scheme)

Time : 3 Hours Maximum Marks : 100

$(5 \times 20 = 100)$

I. (a) State Legrange's mean value theorem and prove that $\log(1+x) = \frac{x}{1+\theta x}$ where $0 < \theta < 1$ and hence deduce that $\frac{x}{1+x} < \log(1+x) < x,\ x > a$.

(b) Evaluate $\lim_{x \to 0} \frac{xe^x - \log(1+x)}{x^2}$.

OR

II. (a) Show that the radius of curvature at any point of the cycloid $x = a(\theta + \sin\theta),\ y = a(1 - \cos\theta)$ is $4a \cos \theta/2$.

(b) If $y = (\sin^{-1})^2$ Show that $(1-x^2) y_{n+2} - (2n+1) x y_{n+1} - n^2 y_n = 0$.

III. (a) State and prove Euler's theorem on homogenous functions.

(b) If $r^2 = (x-a)^2 + (y-b)^2 + (z-c)^2$ then prove that $\frac{\partial^2 r}{\partial x^2} + \frac{\partial^2 r}{\partial y^2} + \frac{\partial^2 r}{\partial z^2} = \frac{2}{r}$.

OR

IV. (a) If $u = x^2 + y^2 + z^2, v = xy + yz + zx, w = x + y + z$ then find $\frac{\partial(u, v.w)}{\partial(x, y, z)}$.

(b) Discuss the maxima and minima of $f(x,y) = x^3 y^2 (1 - x - y)$.

V. (a) Derive the standard equation of parabola.

(b) Find the condition that the line $y = mx + c$ touch the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

(c) Find the equation of the asymptotes of the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$.

OR

(P.T.O.)

VI. (a) Find the locus of the point of intersection of perpendicular tangents to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

(b) Find the equation of hyperbola whose directrix is $2x + y = 1$ and focus (1,2), eccentricity is $\sqrt{3}$.

VII. (a) Find the reduction formula for $\int \cos^n x \, dx$.

(b) Find the area and perimeter of cardiod $r = a(1 - cos\ \theta)$.

OR

VIII. (a) Change the order of integration in $\int_0^a \int_y^a \frac{x}{x^2+y^2}\, dxdy$ and hence evaluate the same.

(b) Evaluate $\int_0^1 \int_0^{\sqrt{(1-x^2)}} \int_0^{\sqrt{(1-x^2-y^2)}} xyz \, dxdydz$.

(c) Calculate the volume by double integration bounded by cylinder $x^2+y^2=4$ and planes $y + z = 4,\ z = 0$.

IX. (a) Prove that $curl\ (grad\ \emptyset) = 0$.

(b) Prove that $\nabla r^n = n(n+1)r^{n-2}$.

(c) Find the directional derivative of $f(x,y,z) = xy^3 + yz^3$ at the point $(2, -1, 1)$ in the direction of vector $i + 2j + 2k$.

OR

X. (a) Prove that $div\left(r^n \bar{r}\right) = (n+3)r^n$.

(b) Explain linearly independent and dependent vectors with examples.

(c) Find the sides of triangle whose vertices are $i - 2j + 2k, 2i + j - k, 3i - j + 2k$.

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